Outage-Based Rate Maximization in CDMA Wireless Networks

M. D'Angelo, C. Fischione, M. Butussi, A. Pinto, A. Sangiovanni-Vincentelli

***Abstract*—The problem of maximizing the sum of the transmit rates while limiting the outage probability below an appropriate threshold is investigated for networks where the nodes have limited processing capabilities. We focus on CDMA wireless network whose rates are characterized under mixed Rayleigh-lognormal fading. The outage probability is given implicitly by a complex function so that solving the optimization problem requires substantial computing. In this paper, we propose a novel explicit approximation of this function that allows solving the problem in an affordable manner. We propose two solutions of the maximization problem with the simplified outage probability constraint: one solves the problem using mixed integer-real programming. The other relaxes the constraints that rates be integers yielding a standard convex programming optimization that can be solved much faster. Numerical results show that our approaches perform well for average values of the outage requirements.**

***Index Terms*—Radio Power Control, CDMA, Outage, Combinatorial Optimization.**

I. Introduction

THE advent of multimedia services over wireless ad-hoc communication networks is increasing the demand of large transmission capacity. Being able to maximize the amount of data (or throughput) that can be transmitted with a given communication quality plays a fundamental role for typical services provided by these networks, as video monitoring, surveillance and industrial automation [1]. Since these networks have limited processing capabilities, algorithms that allow computing quickly the maximum throughput are an essential design tool.

In this paper, we focus on Code Division Multiple Access (CDMA) ad-hoc networks. In CDMA, the transmit rate of each node of the network can be adapted to the wireless channel conditions and traffic load by controlling the radio transmit powers such that Multi-Access Interference (MAI) caused by co-channel transmitters is kept within acceptable levels. Maximizing the transmit rates by power control can be cast as an optimization problem, where the objective function is the sum of the rates of all transmitters, and the constraints are expressed in terms of quality of service.

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In this paper, we propose a method to maximize the throughput for resource-constrained CDMA wireless systems by allocating powers and rates. We focus on delay-limited services, such as video transmission, where the quality of the communication is expressed by outage probability constraints [2], [3]. The rate maximization problem is complex, because of the relations among outage probability, radio powers and transmit rates. In addition, in many wireless propagation scenarios, no-closed form expression is available for the outage probability. Consequently, since the constraints are implicit functions that must be solved at each iteration, the complexity of the optimization approach is out of reach for networks with limited computing capabilities. Our approach is then based on approximating the expression for outage probability with an explicit function trying to maintain accuracy.

The rate maximization problem is similar to radio power control with outage constraints. Minimizing of the powers under outage constraints was investigated in [4]. In [5], the authors considered lognormal fading channels with a Gaussian approximation for the computation of the outage probability. This approach was further developed in [6], where the channel was modeled with Rayleigh fading distributions. Both [5] and [6], showed that power control under outage constraints can be cast as Geometric Program. In [7], the constraints were relaxed by an upper bound provided by the Jensen's inequality. In [8], some of the authors of the present paper introduced a complete framework to solve the rate maximization problem with outage constraints in CDMA systems. Specifically, the channel was modeled by a general Nakagami-lognormal distribution. The outage probability constraints were approximated by resorting to a Wilkinson moment matching approach, and the rate optimization problem was solved by a mixed integer-real problem.

All the contributions mentioned above can be hardly applied to rate maximization on computing resource constrained wireless networks, because of the computational complexity required to compute the solution. In this paper we extend the work in [8] to the case of computationally constrained wireless networks. Our original contribution is twofold: First, we propose a linear approximation for the outage probability in the general context of Rayleigh-lognormal fading. Such an approximation is used in the context of the branch-and-bound method proposed in [8], where the transmit rates are integer values. Then, we relax the constraint on rates to be real values so that there are no integer variables in the optimization problem, which becomes a convex mathematical programming problem. We show that these new approaches are computationally attractive and their accuracy for the outage probability is adequate.

The paper is organized as follows: We present the system model and the mathematical formulation of the problem in Section II. In Section III we describe the approximation of the outage constraint. In Section IV-A we formulate the problem in term of a mixed integer-real programming, whereas in Section IV-B we relax the integrality constraints on the rates and formulate the resulting optimization problem as a convex mathematical programming problem. Numerical results are reported in Section V to demonstrate the properties of our approach.

II. Problem formulation

We consider a system scenario where K mobile transmitters communicate towards a receiver. Each transmitter $i = 1, \ldots, K$ is associated with a traffic source type (voice, video, data, etc.), a level of radio transmit power P_i, and a chip time T_c. Let the channel gain of signals of transmitter i be h_i. We adopt the Lee and Yeh multiplicative model [9, pag. 91]: $h_i = l_i z_i \Omega_i$, where l_i is the path loss, z_i is the fast fading, and Ω_i is the shadowing. The fast fading z_i is modelled as the square of a Rayleigh-distributed random variable. For any $i \neq j$, z_i and z_j are considered statistically independent. The shadow fading is $\Omega_i = \exp(\xi_i)$, where ξ_i is a Gaussian random variable having zero mean and standard deviation σ_{ξ_i}. Let the binary random variable ν_i be the activity status (on/off) of the source. The probability mass function is such that $\Pr[\nu_i = 1] = \alpha_i$ and $\Pr[\nu_i = 0] = 1 - \alpha_i$, where α_i is the activity factor of source i. Let $\mathbf{h} = [h_1, \ldots, h_K]^T$ and $\boldsymbol{\nu} = [\nu_1, \ldots, \nu_K]^T$. Independence is assumed between any pair of variables of the vectors $\mathbf{h}$ and $\boldsymbol{\nu}$. The model of the physical layer of an asynchronous binary phase shift keying CDMA system is summarized by the following expression of the Signal to Interference plus Noise Ratio (SINR) [10]:

$$\mathrm{SINR}_i(\mathbf{h}, \boldsymbol{\nu}) = \frac{P_i h_i}{\frac{N_0}{2T_i} + \Psi_i(\mathbf{h}, \boldsymbol{\nu})} ,$$

where

$$\Psi_i(\mathbf{h}, \boldsymbol{\nu}) = \sum_{\substack{j=1 \\ j \neq i}}^{K} \frac{P_j}{G_i} h_j \nu_j , \tag{1}$$

We model the rate of each transmitter as $R_i = R_{i0} n_i$, where $R_{i0} = 1/T_{i0}$ is the basic rate (with basic bit time T_{i0}), and n_i is an integer denoting the assigned rate. Such a rate is a power of two due to the spreading code structure [9]. Consequently, the spreading factors are expressed as $G_i = G_{i0}/n_i$, where $G_{i0} = T_{i0}/T_c$ corresponds to the basic rate (when $n_i = 1$). We assume that the transmit radio power is expressed as $P_i = p_i n_i$, where p_i is the power at the basic rate. Let $\mathbf{p} = [p_1, \ldots, p_K]^T$, $\mathbf{p_{-i}} = [p_1, p_2, \ldots, p_{i-1}, p_{i+1} \ldots p_K]^T$, $\mathbf{n} = [n_1, n_2, \ldots, n_K]^T$, and $\mathbf{n_{-i}} = [n_1, n_2, \ldots, n_{i-1}, n_{i+1}, \ldots n_K]^T$.

The objective function of the rate maximization problem is the sum of the transmitters' rates, and the constraints are bounds on the maximum value for the outage probability, transmit powers and rates:

$$\begin{aligned}
\mathcal{P}: \quad & \max_{\mathbf{p},\mathbf{n}} \ \mathbb{1}^T \mathbf{n} \\
\text{s.t.} \quad & \Pr[\mathrm{SINR}_i(\mathbf{h}, \boldsymbol{\nu}) < \gamma_i] \leq P_{\mathrm{out}} , \quad \forall i = 1, \ldots, K \\
& \mathbf{p}^T(\mathbf{n} \circ \mathbf{l}) \leq P_T , \\
& p_i \geq p_{i0} , \quad \forall i = 1, \ldots, K \\
& 1 \leq n_i \leq G_{i0}, \quad n_i \in \mathcal{N}, \quad \forall i = 1, \ldots, K
\end{aligned}$$

where $\mathbb{1}^T = [1 \quad 1 \ldots 1] \in \mathbb{R}^K$, and $\mathbf{l}^T = [l_1 \quad l_2 \ldots l_K]$. The decision variables are the powers $\mathbf{p}$ and rates $\mathbf{n}$. P_{out} is the maximum allowed outage probability of the SINR for user i with respect to the threshold γ_i. The constraint $\mathbf{p}^T(\mathbf{n} \circ \mathbf{l}) = \sum_{i=1}^{K} n_i p_i l_i \leq P_T$ is due to the fact that the antenna of the receiver can accept only a maximum amount of power without distorting the signal. For obvious physical reasons, powers cannot be smaller than a given value p_{i0}. The rate constraint is motivated by that the spreading factor is $G_i = G_{i0}/n_i \geq 1$. Finally, the set of rates, $\mathcal{N}$, must be a power of two.

Solving Problem $\mathcal{P}$ is not easy: The outage constraints are complicated functions of rates and power, and the rates are integers. We approach these issues in the next sections.

III. Outage probability approximation

In this section, we propose to express the outage probability constraints as linear functions of the decision variables of Problem $\mathcal{P}$.

The outage probability is obtained by the probability distribution function (pdf) of the SINR, which, however, is unknown for Rayleigh-lognormal fading. A numerical calculation of the outage probability can be found, e.g., in [11]. However, the approach is computationally prohibitive for optimization purposes. An accurate approximation based on the Wilkinson moment matching was proposed in [8]. However, this approximation is still a highly non-linear function of the powers and rates that requires significant computing power.

Let us rewrite the SINR as follows:

$$\mathrm{SINR}_i(\mathbf{h}, \boldsymbol{\nu}) = \frac{p_i l_i z_i \exp(\xi_i)}{\frac{N_0}{2G_{i0}T_c} + \Psi_i(\mathbf{h}, \boldsymbol{\nu})} . \tag{2}$$

The new approximation of the outage probability is obtained as follows. First, $\Psi_i(\mathbf{h}, \boldsymbol{\nu})$ is approximated by its average. As observed in [12], taking the average of the MAI to compute the outage probability corresponds to considering that the fluctuations of the MAI term may have a variance that is small when compared to the variance of the user signal power. Second, the product between $\exp(\xi_i)$ and z_i in the SINR expression can be approximated by a lognormal variable $\exp(\hat{\xi}_i)$ as proposed in [9, pag. 92]. Combining these two, we obtain

$$\mathrm{SINR}_i(\mathbf{h}, \boldsymbol{\nu}) \approx \frac{1}{g_i(\mathbb{E}\,\Psi_i(\mathbf{h}, \boldsymbol{\nu}))} \exp \hat{\xi}_i , \tag{3}$$

where $\hat{\xi}_i$ is a Gaussian random variable with mean and standard deviation, respectively, as

$$\begin{aligned}
\mu_{\hat{\xi}_i} &= \psi(1) , \\
\sigma^2_{\hat{\xi}_i} &= \zeta(2, 1) + \sigma^2_{\xi_i} ,
\end{aligned}$$

where $\psi(1)$ is Euler's psi function, and $\zeta(2,1)$ is Riemann's zeta function, as defined in [9, pag. 107], whereas

$$g_i(\mathbb{E}\,\Psi_i(\mathbf{h},\boldsymbol{\nu})) = \frac{1}{p_i l_i}\left(\frac{N_0}{2G_{i0}T_c} + \mathbb{E}\,\Psi_i(\mathbf{h},\boldsymbol{\nu})\right)$$

and

$$\mathbb{E}\,\Psi_i(\mathbf{h},\boldsymbol{\nu}) = \sum_{\substack{j=1\\ j\neq i}}^{K} \frac{p_j n_j}{G_{i0}} l_j \mu_{z_j}\alpha_j \exp\frac{1}{2}\sigma^2_{\xi_j}\,. \tag{4}$$

From (3), and since $\hat{\xi}_i$ has Gaussian distribution,

$$\begin{aligned}&\Pr\left[\mathrm{SINR}_i(\mathbf{h},\boldsymbol{\nu}) < \gamma_i\right]\\ &\approx 1 - Q\left(\frac{\ln\gamma_i + \ln g(\mathbb{E}\,\Psi_i(\mathbf{h},\boldsymbol{\nu})) - \mu_{\hat{\xi}_i}}{\sigma_{\hat{\xi}_i}}\right).\end{aligned} \tag{5}$$

where $Q(x) = 1/\sqrt{2\pi}\int_x^\infty e^{-t^2/2}dt$ is the complementary cumulative Gaussian distribution.

Using (4) – (5), the constraints on the outage probability can be rewritten as

$$\frac{p_i}{I_i(\mathbf{n}_{-i},\mathbf{p}_{-i})} \geq \gamma_i \quad \forall i = 1,\ldots,K\ , \tag{6}$$

where $I_i(\mathbf{n}_{-i},\mathbf{p}_{-i})$ is defined as follows:

$$\begin{aligned}I_i(\mathbf{n}_{-i},\mathbf{p}_{-i}) = &\left(\frac{N_0}{2l_iG_{i0}T_c} + \frac{\mathbb{E}\,\Psi_i(\mathbf{h},\boldsymbol{\nu})}{l_i}\right)\\ &\cdot\exp(-\sigma_{\hat{\xi}_i}q_i - \mu_{\hat{\xi}_i}),\end{aligned} \tag{7}$$

with

$$q_i = Q^{-1}(1 - P^i_{\mathrm{out}})\,.$$

The function (7) is called the interference function. It is a quadratic function of rates and powers. By contrast, the approximation proposed in [8] is a highly non linear function. The interference function we have proposed in this paper enables us the development of computationally efficient algorithms for the solution of Problem $\mathcal{P}$.

IV. Problem Solution

In this section, we use in Problem $\mathcal{P}$ the outage probability approximation developed in the previous section, and present two strategies for the solution of the problem. The first one makes use of a branch-and-bound method, whereas the second one, by relaxing the constraints on rates to be integer, transforms the problem into a convex optimization one.

A. Relaxation 1

Equation (7) allows rewriting the optimization problem $\mathcal{P}$ as follows:

$$\begin{aligned}\mathcal{P}_1 :\quad &\max_{\mathbf{n},\mathbf{p}}\ \mathbb{1}^T\mathbf{n}\\ \text{s.t}\quad &\frac{p_i}{I_i(\mathbf{n}_{-i},\mathbf{p}_{-i})} \geq \gamma_i \quad \forall\, i = 1,\ldots,K\\ &\mathbf{p}^T(\mathbf{n}\circ\mathbf{l}) \leq P_T\\ &p_i \geq p_{i0} \quad \forall\, i = 1,\ldots,K\\ &1 \leq n_i \leq G_i,\quad n_i \in \mathbb{N}^+ \quad \forall\, i = 1,\ldots,K\end{aligned}$$

This problem is a mixed integer-real optimization problem. It can be solved by the branch-and-bound approach described in [8], which is based on the availability of feasible power vectors. In particular, from Proposition 1 in [8], these power vectors can be evaluated by the following optimization problem:

$$\begin{aligned}\mathcal{P}_2 :\quad &\min_{\mathbf{p}}\ \mathbf{n}^T\mathbf{p}\\ \text{s.t.}\quad &\frac{p_i}{I_i(\mathbf{n}_{-i},\mathbf{p}_{-i})} = \gamma_i \quad \forall\, i = 1,\ldots,K\end{aligned}$$

Note that in this problem the decision variable is only $\mathbf{p}$, whereas the rate vector is fixed. The solution of Problem $\mathcal{P}_2$ was obtained in [8] by iterative numerical methods. In this paper we can exploit the function $I_i(\mathbf{n}_{-i},\mathbf{p}_{-i})$ derived in (7) to simplify the solution. Since this function is linear in $\mathbf{p}_{-i}$, there is a unique optimal solution of Problem $\mathcal{P}_2$, which is given just by the solution of the system of the linear equations given by the constraints. After some manipulations, the optimal solution of Problem $\mathcal{P}_2$ is found to be

$$p_i = \frac{c}{\sum_{\substack{l=1\\ l\neq i}}^{K}\frac{b_l n_l}{a_l + b_l n_l} - a_i}\,, \tag{8}$$

where:

$$a_j = G_{i0} l_i \phi_i\,, \tag{9}$$

$$b_j = l_j \mu_{z_j}\alpha_j \exp\left(\frac{1}{2}\sigma^2_{\xi_j}\right)\,, \tag{10}$$

$$c = -\frac{N_0}{2T_c}\,, \tag{11}$$

with

$$\phi_i = \frac{1}{\gamma_i}\exp(\sigma_{\hat{\xi}_i}q_i + \mu_{\hat{\xi}_i})\,.$$

The closed-form expression of the feasible power vectors (8) is computationally simple to evaluate. On the contrary, the optimal solution presented in [8] is achieved by numerical iterations, and hence is computationally substantially more expensive. On the other hand, the availability of a closed-form power vectors comes at the price of reduced accuracy of the outage constraints.

B. Relaxation 2

So far, we have investigated the solution of problem $\mathcal{P}$ where the rates are integer values. If we relax the integrality constraint on the rates, and consider the approximation of the outage probability as we have proposed in Section III, we can cast the rate maximization problem as a convex optimization problem. Rewriting the problem $\mathcal{P}_1$ according to the canonic form [13], we obtain that

$$\begin{aligned} \mathcal{P}_3: \quad & \min_{\mathbf{n},\mathbf{p}} -\mathbb{1}^T\mathbf{n} \\ \text{s.t.} \quad & \sum_{\substack{j=1\\ j\neq i}}^{K} b_j p_j n_j - a_i p_i - c \le 0 \quad \forall\, i = 1,\ldots,K \\ & \mathbf{p}^T(\mathbf{n}\circ\mathbf{l}) \le P_T \\ & p_{i0} - p_i \le 0 \quad \forall\, i = 1,\ldots,K \\ & 1 - n_i \le 0 \quad \forall\, i = 1,\ldots,K \\ & n_i - G_i \le 0 \quad \forall\, i = 1,\ldots,K \end{aligned}$$

where the outage constraint has been rewritten using (7), and a_i, b_i and c are defined in (9), (10), and (11), respectively. The cost function and all constraints of Problem $\mathcal{P}_3$ are convex. A feasible solution of Problem $\mathcal{P}_3$ is given by all zero radio powers and minimum transmit rates. Therefore, a global optimal solution exists, which can be obtained by efficient interior point methods.

V. Numerical Results

In this section we evaluate the accuracy of the approximation of the outage probability proposed in Section III, and then we solve the optimization problem as proposed in Sections IV-A and IV-B. The numerical results obtained in this section are compared with the approach proposed in [8].

The system parameters are taken from the 3GPP specification [14]: the chip time is $T_c = 2.6\times10^{-7}$s, and the maximum spreading factor is $G_{i0} = 256$. We assume a thermal noise $N_0 = -170$ dBm/Hz, and we set $P_T = 1.5239 \times 10^{-12}$, such that $P_T T_c/N_0 = 16$ dB. We assume a homogeneous propagation environment for the path loss l_i, which is set to 90 dB for each transmitter. As required for the validity of the multiplicative model of the fading, μ_z is set to 1 [9]. The system is considered in outage when the SINR is below $\gamma_i = 3.1$. All transmitters have the same activity factor $\alpha_i = 0.5$.

A. Approximation of the Outage Probability

To establish the accuracy of the proposed approximation, we performed Monte Carlo simulations. In particular, we considered (2) and collected a very large set of SINR samples by drawing the random variables ξ_i and z_i. The probability distribution function of the SINR was then evaluated and compared with the approximation proposed in Section III. Our approximation is assessed by the percentage difference between the real ccdf of the SINR and the approximated ccdf. This provides an evaluation of the percentage error that affects the $Q(\cdot)$ term in (5).

For each simulation, we considered different scenarios in terms of number of transmitters (K) and shadowing variance (σ_{ξ_i}). Each transmitter used a radio power of -10 dBm at the basic rate. The range of SINR thresholds γ being considered is the one typically adopted for the outage computation [9].

The plots in Fig. 1 report the ccdf percentage error of our approximation for a scenario with $K = 8$ transmitters, whereas Fig. 2 reports the ccdf percentage error of our approximation for a scenario with $\sigma_{\xi_i} = 0.5$, for $i = 1,\ldots,K$. These figures are referred to one of the transmitters considered. Other transmitters exhibit similar behaviors. The approximation achieves a good accuracy for small value of γ, with an error which is usually below 2 %. A larger error characterizes the scenarios with a higher variance of the MAI ($\Psi_i(\mathbf{h},\boldsymbol{\nu})$), which, by (1), is an increasing function of K and σ_{ξ_i}. The error shows a larger sensitivity to the number of transmitters in the system. Even in the case of 12 transmitters, however, the error is still below 2% for values of the SINR less than 8dB. Notice that the approximation overestimates the actual ccdf for low γ values, while it provides an underestimation for higher values.

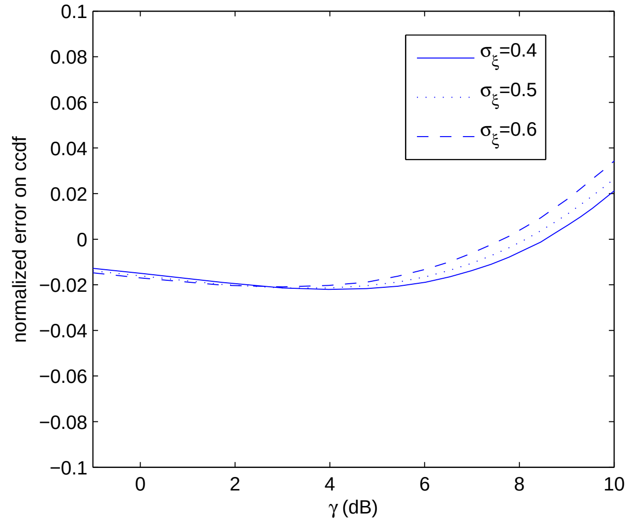

Fig. 1. Complementary cumulative distribution function percentage error of our approximation for a scenario with $K = 8$ transmitters. Each curve refers to a different shadowing condition (σ_{ξ_i}).

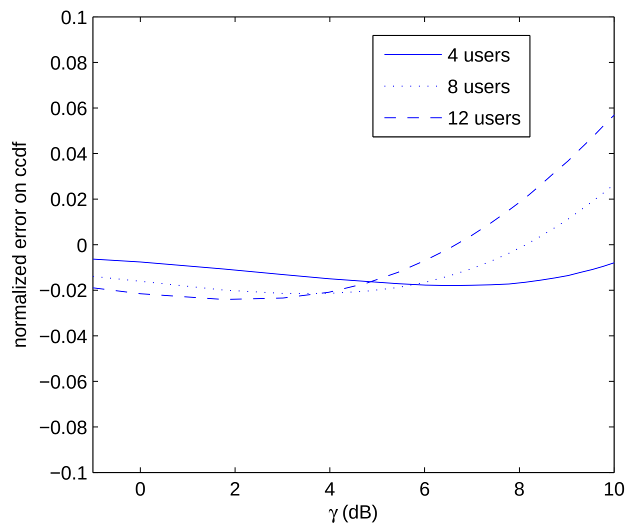

Fig. 2. Complementary cumulative distribution function percentage error of our approximation for a scenario with $\sigma_{\xi_i} = 0.5$. Each curve refers to a system with a different number of transmitters (K).

B. Optimal Rate

The plots in Figs. 3 – 5 show the maximum rate sum that can be achieved in a scenario with 4, 8 and 12 transmitters, respectively. We assumed that $\sigma_{\xi_i} = 0.5$ for each transmitter. Other values of σ_{ξ_i} provide the same trend in the numerical results, as those discussed below. Each plot reports the solution of problem $\mathcal{P}$ as obtained by the proposed approaches, namely:

- Relaxation 1 are the result obtained by the mixed integer-real programming $\mathcal{P}_1$, where the approach proposed in III is used to approximate the outage constraint (see Subsection IV-A);
- Convex 2^N are the result obtained by convex programming $\mathcal{P}_3$ (see Subsection IV-B), and then mapping the rates in $2^{\mathbb{N}}$; the mapping simply consists of taking the $\log_2(\cdot)$ of $n \in \mathbb{R}^+$, and flooring the result to the nearest integer;
- Relaxation 2 are the result obtained by the convex programming $\mathcal{P}_3$, with $n \in \mathbb{R}^+$ (see Subsection IV-B);
- Wilkinson approximation are the result obtained by the mixed integer programming $\mathcal{P}_1$, where the Wilkinson moment matching approach is used to approximate the outage constraint as proposed in [8];

In Figs. 3 – 5, the curves obtained by the Wilkinson approximation in [8] are the benchmark to assess the performance of the methods proposed in this paper. In the figures, a zero rate means that the rate maximization problem is not feasible with respect to the given constraints and configuration.

Comparing the results of the mixed integer-real programming with the different approximations, we observe that the solutions proposed in this paper are close for intermediate ranges of the outage requirements P_{out}. Furthermore, by using the Relaxation 2 along with a simple mapping of the relaxed rates in $2^{\mathbb{N}}$, taking the $\log_2(\cdot)$ and rounding to the nearest integer (floor), we obtain a rate sum that is equal or slightly smaller than Relaxation 1 result. The conclusion is that our solutions work very well for intermediate values of the outage constraints. Notice that in all the scenarios there is always a cross-point between the curves for certain values of the outage requirements.

For outage requirements lower than the crossing point, our solutions provide an over-allocation of the transmit rates. For instance, looking at Fig. 3, the transmitters use rates higher than the one obtained by using the Wilkinson approximation. Computing the outage probability with such a rate allocation, a probability of outage of 0.01 cannot be achieved: the rates and powers obtained by solving the problem determine a $P_{\text{out}} \approx 0.025$, according to the actual distribution of the SINR, whereas the requirement was 0.01. The same considerations apply to the case of the solutions for the scenario with 8 transmitters (Fig. 3), where a probability of outage less than 0.04 gives an actual $P_{\text{out}} \approx 0.05$.

For higher values of the outage probability requirement, the proposed solutions underestimate the achievable rates, particularly for very high outage requirements. Obviously, an under allocation of the rates as a consequence of the approximation guarantees the satisfaction of the actual outage constraints.

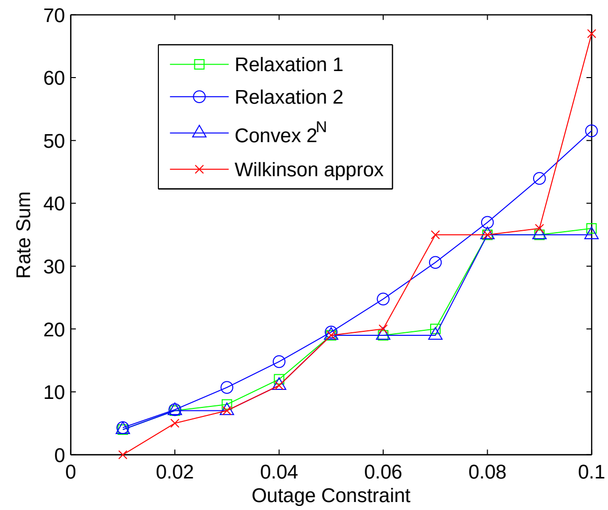

Fig. 3. Optimal Rate Sum for a scenario with $K = 4$ transmitters and $\sigma_{\xi_i} = 0.5$, for $i = 1, \ldots, K$.

C. Computational Complexity

The computational complexity of the rate maximization problem investigated in this paper is defined as the number of iterations necessary to obtain the optimal solution. In each iteration, a set of closed-form expressions must be evaluated. The performance of the solutions proposed in this paper are less computationally expensive than the method in [8], which counterbalance the loss of accuracy achieved for very low and very high outage constraints.

Relaxation 1 is about 5 to 10 times less computationally complex than the method presented in [8], because the outage approximation proposed in this paper allows the solution of Problem $\mathcal{P}_2$ in closed form. From Monte Carlo simulations, we observed that the the number of iterations necessary to obtain the optimal solution is less than 335 for 4 transmitters, 6070 for 8 transmitters and 7100 for 12 transmitters, respectively.

Relaxation 2 is the least computational expensive. From Monte Carlo simulations, we have observed that the solution is achieved in about 200 iterations in each scenario. However, recall that Relaxation 2 is the least accurate solution for very high and very low outage requirements.

VI. Conclusions

We investigated a simple approximation of the Signal to Interference + Noise Ratio of CDMA systems for the computation of the outage probability. Then, we applied this approximation to the rate maximization problem under outage constraints. We proposed two approaches to solve the simplified problems and investigated the quality of the optimal solutions of the simplified problems against the accurate one provided in [8].

The methods investigated in this paper require a reduced computational load, which is particularly useful for networks with limited processing capabilities at the potential expense of quality of the solution due to the approximations used. Numerical results show that our approximation performs well

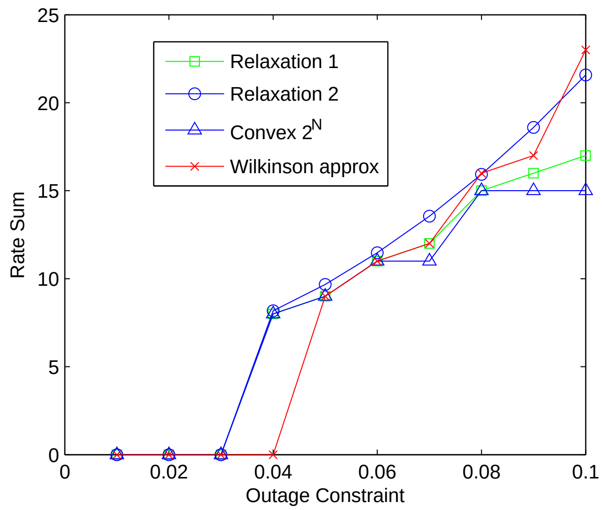

Fig. 4. Optimal Rate Sum for a scenario with $K = 8$ transmitters and $\sigma_{\xi_i} = 0.5$, for $i = 1, \ldots, K$.

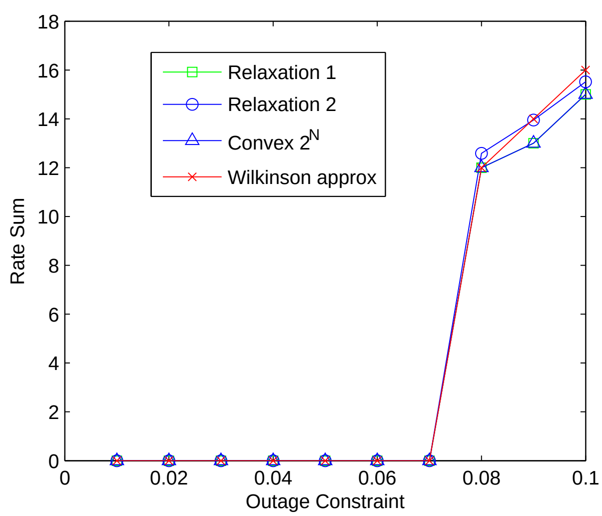

Fig. 5. Optimal Rate Sum for a scenario with $K = 12$ transmitters and $\sigma_{\xi_i} = 0.5$, for $i = 1, \ldots, K$.

for low values of the outage requirements. It underestimates the actual probability with a maximum error that is less than 2% in situations of practical interest. The sub-optimal solutions obtained with the approximation perform well for average values of the outage requirements.

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