Correct-by-Construction and Optimal Synthesis of Beacon-Enabled ZigBee Network

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***Abstract*—In this paper we develop a formal approach for the synthesis of a cost-effective and correct-by-construction communication network (focusing on ZigBee wireless networks) subject to a set of end-to-end communication constraints of latency, bandwidth and error-rate, together with the constraints of the network protocols and the desired geographical placement of the network. We also develop a software platform to implement the proposed approach for network synthesis, and apply it to a practical wireless network synthesis for centralized as well as distributed estimation application.**

***Note to Practitioners*—Network synthesis begins by specifying a set of point-to-point quality-of-service requirements of latency, throughput and error-rate. For a ZigBee-based wireless network (one of the popular networks), we present a mathematical approach to its synthesis for a given set of service specifications along with the geographical placement. The approach formalizes the synthesis problem that can be adopted for other types of networks, and guarantees its correctness as well as optimality.**

***Index Terms*—Building automation, integer linear programming, network synthesis, ZigBee.**

I. Introduction

IN THIS PAPER, we study the synthesis of a wireless network with time-triggered medium-access, in particular, the ZigBee network in its beacon-enabled (synchronized) mode.

Our work on communication synthesis starts from a description of the communication requirements in terms of connectivity and quality-of-service (QoS). These requirements are derived by control engineers from the control performance requirements such as stability, steady-state error, and settling time.

The fact that the requirements of sampling rate and measurement accuracy (which depends on quantization-accuracy as well as channel-reliability) impose certain end-to-end constraints on the communication network has been formalized in the literature. For example, [15] shows that a sufficient condition for stabilizing a linear system, under control over a communication channel, is that the packet-size times the success-rate (which is the capacity of a lossy-channel) exceeds the sum of the logarithms of the magnitudes of the unstable eigenvalues. The communication network's end-to-end constraints include maximum latency (which must not exceed the sampling-period), minimum packet-size (to ensure a minimum quantization accuracy), and a maximum error-rate (to ensure a maximum signal-distortion). Furthermore, additional constraints arise due to a desired geographical placement of the network (router and link locations), and also other performance constraints may exist (such as maximum utilization). Since the cost of a communication network constitutes a large portion of the overall networked control/embedded system, care needs be taken in optimizing the network setup and operating cost subject to the aforementioned constraints.

Motivated by this, in our previous works [12], [13], we have been developing an approach for cost-effective and correct-by-construction communication network synthesis (focusing on ZigBee networks). The network synthesis issues of router-placement and connection-routing are formulated as an instance of an Integer Linear Programming (ILP) problem. In [12], the scheduling of connections was assumed given. However, the scheduling of connections is needed to help synchronize the nodes, and also to implement the precedence constraints introduced by the routing decisions. In [13], we extended our work in [12] by including scheduling of connections as part of the network synthesis problem. In this paper, we refine the scheduling constraints of [13] to accurately capture the restrictions of the wireless network protocol (ZigBee in the present case).

Router placement and scheduling has also been addressed in the following works. Reference [3] studied the problem of placing routers so as to guarantee full network connectivity and router redundancy for robustness. A simulated annealing approach to router placement to maximize network connectivity and user coverage was proposed in [16]. In these works, the quality of service issues of communication delay, bandwidth (throughput), etc., were not taken into consideration. Also, the cost of the resulting router placement and the constraints on the resulting scheduling were not considered. The integrated optimal routing, scheduling and power control was addressed in [7], where the goal was to minimize the total average transmission power subject to the minimum average data rate per link, and peak transmission power per node. The end-to-end communication constraints were not considered (only per link). The study on joint router placement and link scheduling was also reported in [5], where the authors proposed a column generation-based heuristic approach (for a Mixed ILP formulation) to assign the required number of time slots to the links such that the bandwidth requirement can be satisfied. The problem

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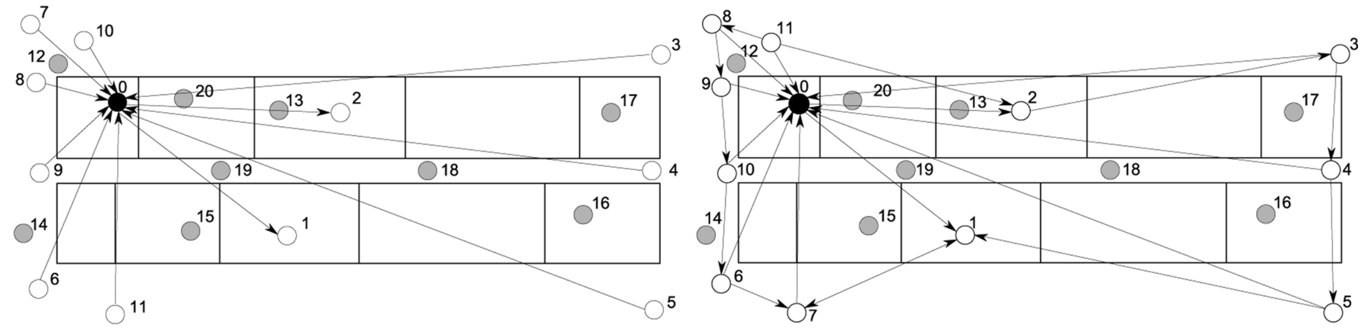


Fig. 1. Application of network synthesis.

of placing as few as possible gateways to support the required bandwidth and throughput based on ILP was studied in [4], and to support a fair bandwidth allocation based on Mixed ILP was studied in [8]. In our work, we take into account the constrains of tree-based network topology on scheduling, and we also consider the end-to-end communication delay, geographical constraints of router placements, and cost of the synthesized network.

In the approach we consider, we attempt to meet the QoS specifications without having to do packet retransmissions. While such retransmissions can help to meet the packet error rate requirement, it adds additional delays that are not conducive to real-time applications where the latency requirement is a key. Finally, the current synthesis approach is limited to the stationary nodes. Many applications, including the one that we consider do not at present use mobile nodes. Also, in a real environment, any change in the network health causes the node availability and the path-losses to change. If the capability to monitor network health is available, our optimization formulation can be reexecuted to compute possibly new placement, routing and scheduling decisions.

In order to illustrate our approach, we apply it to a practical-sized wireless network synthesis problem for centralized as well as distributed estimation in a building automation application of which the connectivity requirements are shown in Fig. 1. The details are given in Section IV.

II. Overview of ZigBee Network

At the physical layer, the IEEE 802.15.4 standard is used that offers a total of 27 channels, with a peak data-rate of 250 Kbits/s. At the MAC layer, nodes are grouped into Personal Area Networks (PANs). A PAN is started by a router node, which assumes the role of PAN Coordinator. Other router and non-router (end-device) nodes, called children nodes, associate with the existing routers of a PAN, called parents, to form a larger PAN. Nodes within a PAN operate on their own communication frequencies; the interchannel interference if any is accounted in our signal-to-interference-and-noise ratio (SINR) formula of Section III. Nodes within a PAN may be synchronized (beacon-enabled mode) to communicate in a time-division multiple-access (TDMA) mode.

To manage synchronized medium access, superframe structures consisting of active and inactive periods, as shown in Fig. 2, are used. Active/inactive periods of different

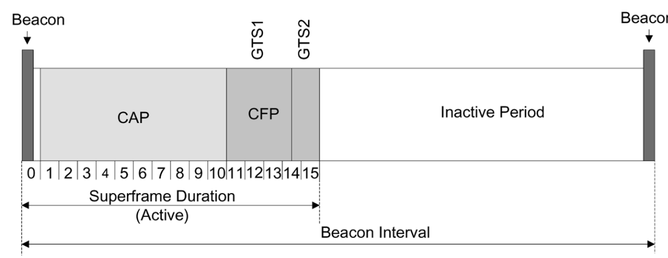


Fig. 2. ZigBee network superframe structure.

routers within a PAN are phased so as to avoid overlap by beacon-scheduling. Beacon-scheduling is an active research area [6], [10], [17], and our synthesis algorithm automatically ensures non-overlapping scheduling, complementing the above works.

In a superframe, the time interval between two consecutive beacons is called Beacon Interval (BI) and is defined as $BI = aBaseSuperframeDuration \times 2^{BO} symbols$, where BO represents Beacon Order and $aBaseSuperframeDuration$ has a constant duration of $960 \text{ symbols} = 960 \text{ symbols} \times 4 \text{ bits/symbol} \times 1/(250 \text{ K}) \text{ secs/bit} = 15.36 \text{ msec}$, corresponding to the minimum duration of the superframe (case of $BO = 0$). The beacon order BO can range from 0 to 14 ($BO = 15$ means that no beacon will be transmitted, i.e., the non beacon-enabled mode).

The active period of a superframe is called a macroslot (its bit capacity denoted as $B_{\max}$), the duration of which is determined by the Superframe Duration (SD), defined as $SD = aBaseSuperframeDuration \times 2^{SO}$ symbols, where the superframe order SO can range from 0 (minimum-sized macroslot) to BO (maximum-sized macroslot and no inactive period). Superframe structures are shifted in phase by multiples of a SD (so their active periods don't overlap). Thus up to $BI/SD = 2^{BO-SO}$ superframe structures with non-overlapping active periods (macroslots) can fit within a BI; the actual number can be smaller which we denote as $n_{\max}$ in the paper. Superframe Duration SD is divided into 16 equal-sized minislots (bit capacity denoted as $b_{\max}$), classified into Contention Access Period (CAP) and Contention Free Period (CFP). We let the contention free period to span the entire macroslot, and in which case it can consist of up to 16 Guaranteed Time Slots (GTSs). The actual number of GTSs used, denoted $m_{\max}$ in the paper, may be smaller than the maximum allowed. Transmissions in the GTSs are uniquely allocated to devices, and concurrent transmissions by the devices in the same PAN are forbidden.

III. Network Synthesis With Scheduling

A. Network Synthesis Parameters

1) Quality of Service Parameters: The nodes and the connections requirements of a communication network can be specified as a directed graph (N, C) consisting of a set of *nodes* N and a set of *connections* $C \subseteq N^2$. N is further partitioned into $E \cup R$, the set of end devices E and *candidate* routers R. The set of end devices E consists of the sets of source locations S and destination locations D. The network is synthesized to serve a set of connections, i.e., a set of source-destination pairs $C \subseteq S \times D \subseteq N^2$. Each connection $c \in C$ is labeled with a QoS requirement which is a 3-tuple (l_c, b_c, p_c): l_c is the maximum latency (which must not exceed the sampling-period), b_c is the number of bits per message (to ensure a minimum quantization accuracy), and p_c is the maximum packet error rate probability (to ensure a maximum signal-distortion).

2) Path-Loss and Error-Rate: Suppose the node i transmits packets with a radio power level P_i. Let the distance between node i and j be denoted by $d_{i,j}$. We denote the path loss attenuation between the transmitter and the receiver by $\mathrm{PL}(d_{i,j})_{dB}$, satisfying [14]

$$\mathrm{PL}(d_{i,j})_{dB} = \mathrm{PL}(d_0)_{dB} + 10\beta \log_{10}\left(\frac{d_{i,j}}{d_0}\right) + \Omega_{i,j} + \mathrm{PL}_{mw}$$

where $\mathrm{PL}(d_0)$ denotes the path loss computed at a reference distance d_0, β denotes the path loss exponent, $\Omega_{i,j}$ denotes the shadowing attenuation modeled as a Gaussian random variable having zero average and variance $\sigma^2_{i,j}$, and PL_{mw} is the path-loss due to multiple-walls. We adopt a multi-wall model [2] to account for the path loss due to the presence of walls between a transmitter and a receiver: $\mathrm{PL}_{mw} = L + n_w L_w$, where L is a constant, n_w is the number of walls intersected by the line of sight between the transmitter and the receiver, and L_w is a constant depending on the thickness of the wall.

Assuming that the nodes are not simultaneously transmitting (i.e., the network operates in beacon-enabled mode), the formula for power received from node i at node j, $P_{i,j}$, is given by

$$10 \log_{10} P_{i,j} = 10 \log_{10} P_i - \mathrm{PL}(d_{i,j})_{dB}$$

where P_i is power transmitted by node i. SINR in dB is given by

$$10 \log_{10} \mathrm{SNIR}_{ij} = 10 \log_{10} P_{ij} - 10 \log_{10} P_n$$

where P_n is the power of thermal noise, with a typical value of -170 dBm.

The bit error probability of the link from node i to node j can be modeled as

$$p_b(\mathrm{SNIR}_{ij}) \triangleq f_1(\mathrm{SNIR}_{ij})$$

where $f_1(\cdot)$ for the case of O-QPSK modulation with coherent demodulation in a slow Rayleigh-fading environment (corresponding to slow moving objects), which exhibits nonselective behavior both in frequency and time, can be expressed by [14]

$$f_1(\mathrm{SNIR}_{i,j}) \approx \frac{1}{2}\left(1 - \sqrt{\frac{\mathrm{SNIR}_{i,j}}{1 + \mathrm{SNIR}_{i,j}}}\right)$$

Assume that a packet at the data-link layer is composed of O bits of protocol overhead and a payload of b_i bits and the CRC code is always able to detect erroneous packets (see [9] for an experimental support). Then, the packet error rate probability, without any retransmission mechanism, can be modeled by

$$p(i,j) \triangleq f_2(\mathrm{SNIR}_{ij}) = 1 - [1 - p_b(\mathrm{SNIR}_{ij})]^{O+b_i}.$$

B. Network Synthesis Formulated as ILP

In order to synthesize a cost-optimal network to offer the desired QoS for each connection, we need to make the following three decisions: (i) placement of routers; (ii) routing of connections; and (iii) scheduling of connections.

1) Router Placement: The locations of sources and destinations are given as part of the connections specification C, whereas the placement of routers to support the connections needs to be determined. For a practical application, the number of candidate locations can be taken to be finite: Based on the communication range of routers, the area required to be connected can be divided into a finite number of zones, where each zone can have at most one router. A binary variable x_i is associated with each candidate location, which equals one if and only if a router is installed at location i. The objective is to place as few and cost-effective routers as necessary.

2) Connection Routing: For each desired connection (source-destination pair), a routing path from source to destination, needs to be determined. For this, we need to determine which pairs of nodes will have active links between them. A binary variable ℓ_{ij} is associated with a pair of nodes, which equals one if and only if a link is installed between i and j, and i is j's parent. A node-link incidence matrix I consisting of elements of 1, -1, and 0 is constructed, where an entry $(k, (ij))$ equals 1 (resp., -1) if node k is a source (resp., destination) of link ij, and 0, otherwise. Then, a route from source s_c to destination d_c of a connection c can be obtained as a solution of a classical balance equation involving the node-link incidence matrix and binary variables y_{ijc} which equals 1 if and only if link ij is used in the route for connection $c \in C$. Let y_c be a vector of size $|N|(|N| - 1)$ obtained by stacking the entries y_{ijc}, and $\mathbf{b}_c$ be a vector of size $|N|$ such that $\mathbf{b}_c(i) = 1$ for $i = s_c \in S$, $\mathbf{b}_c(i) = -1$ for $i = d_c \in D$ and $b_c(i) = 0$ otherwise. Then a solution of $I y_c = \mathbf{b}_c$ provides the values of the decision variables y_{ijc} that ensure the existence of a route (a sequence of links) for a connection $c \in C$. In addition, the routes for the connections must be chosen in such a way that the aggregate error-rate across a route is below the required error-rate of a connection using that route.

The following example illustrates how to construct an incidence matrix and balance equation for a given set of nodes and connections.

3) Example 1: Given three candidate nodes $\{1,2,3\}$, we have $I = \begin{bmatrix} 1 & 1 & -1 & 0 & -1 & 0 \\ -1 & 0 & 1 & 1 & 0 & -1 \\ 0 & -1 & 0 & -1 & 1 & 1 \end{bmatrix}$, where the rows correspond to the three nodes $k \in [1,3]$ and the columns correspond to the six links ℓ_{12}, ℓ_{13}, ℓ_{21}, ℓ_{23}, ℓ_{31}, ℓ_{32}. In I, an entry $(k, \ell_{ij}) = 1$ if $k = i$ (node k is the source of link ℓ_{ij}), $(k, \ell_{ij}) = -1$ if $k = j$ (k is the sink of link ℓ_{ij}), and $(k, \ell_{ij}) = 0$ otherwise. Now, consider a connection $c = (1,3)$, then since node 1 is the source and node 3 is the sink, this connection is captured by a vector $\mathbf{b}_c = (1 \quad 0 \quad -1)'$, where $'$ denotes the transpose operation. A solution of the balance equation $Iy_c = \mathbf{b}_c$, is given by $y_c = (1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0)'$ and corresponds to the route $1 \rightarrow 2 \rightarrow 3$ since it uses the first and fourth rows corresponding to the links ℓ_{12} and ℓ_{23}, respectively.

4) Connection Scheduling: In ZigBee's beacon-enabled mode, the time-line is divided into beacon-intervals, and all communications of a round must fit within a number of beacon-intervals. Each beacon-interval is further divided into macroslots, and each macroslot is assigned to a router so that a subset of its children can communicate data for a subset of connections routed through those children. A router may be assigned multiple macroslots to allow communication of the entire set of connections across the entire set of its children. The reason that all such data may not be routed within a single macroslot is that each connection-route introduces a certain precedence constraint among the nodes of the route, and all different precedence constraints of all the connections must be respected. Moreover, the limitation on the bit capacity of a macroslot as well as of a minislot, and also the limit on the number of minislots must also be respected.

Binary decision variable g_{ijsc} (resp., g'_{ijsc}) is associated with each router i, each node j, and each time-slot s for connection c, which equals 1 if and only if parent i sends (resp., receives) data to (resp., from) child j in macroslot s for connection c.

To summarize, the following decision variables and parameters are used in our ILP formulation of the network synthesis problem.

- Binary decision variables:
  - — x_i: 1 if and only if a device (end device/router) is installed at location i.
  - — ℓ_{ij}: 1 if and only if a link between node i and j is active, and i is j's parent.
  - — y_{ijc}: 1 if and only if the route for connection c uses a link from node i to node j.
  - — z_{isc}: 1 if and only if node i is assigned macroslot s for connection c.
  - — g_{ijsc}/g'_{ijsc}: 1 if and only if parent i sends/receives data to/from child j in macroslot s for connection c (a derived variable that depends on z_{isc} and y_{ijc}).
  - — g_{ijs}/g'_{ijs}: 1 if and only if parent i sends/receives data to/from child j in macroslot s (a derived variable obtained as projection of g_{ijsc}/g'_{ijsc}).
- Network parameters:
  - — E: the set of end devices.
  - — R: the set of routers.
  - — N: the set of nodes, where $N = E \cup R$.
  - — C: the set of connections.
  - — BO: beacon order.
  - — SO: superframe order.
  - — $b_{\max}$: the bit capacity of a minislot.
  - — $B_{\max}$: the bit capacity of a macroslot.
  - — $n_{\max}$: the number of macroslots in one beacon interval, where $n_{\max} \leq 2^{BO-SO}$.
  - — $m_{\max}$: the number of minislots in a macroslot, where $m_{\max} \leq 16$.
  - — $p(i,j)$: the packet error rate probability of link ij.
- Specification parameters:
  - — c_i: the location-i router installation cost.
  - — w_c: the connection-c operation cost per link.
  - — b_c: the number of bits per message of connection c.
  - — p_c: the maximum packet error rate probability of connection c.
  - — l_c: the maximum latency of connection c.

5) Objective Function: The optimization objective for a network is defined as its installation and operation costs: The router installation cost for node i is c_i, whereas the operational cost per connection c, per link it uses, is $w_c = (e_t + e_r)(b_c + O)(T_{life}/BI)(c_B/e_B)$, where $e_t + e_r$ is the energy consumed per bit by transmitter-receiver pair, $b_c + O$ is total number of data and overhead bits for connection c, T_{life}/BI is the total number of rounds of communication in network's life assuming each round fits within one BI, and c_B/e_B is the cost of battery per unit energy stored in the battery.

The proposed ILP formulation for the network synthesis problem is provided next, and is based on the following assumptions as discussed in the introduction.

1) The wireless sensor network to be synthesized is a beacon-enabled ZigBee network.
2) The path loss and battery cost are as captured by the formulae given in Section III.
3) There are no mobile nodes.
4) There are no retransmissions.

The constraints of network synthesis problem can be categorized into four groups: (a) Placement (constraint 1); (b) Routing (constraints 2–9); (c) Scheduling (constraints 10–17); and (d) Quality-of-Service (constraints 18 and 19)

$$\min_{s.t.} \sum\nolimits_{i\in R} c_i x_i + \sum\nolimits_{i,j\in N} \sum\nolimits_{c\in C} w_c y_{ijc}$$

1. $x_i = 1, \forall\, i \in E$.
2. $x_i + x_j \geq 2\ell_{ij}, \forall\, i, j \in N$.
3. $\ell_{ij} + \ell_{ji} \leq 1, \forall\, i, j \in N$.
4. $\ell_{ij} = 0, \forall\, i \in E, j \in N$.
5. $\sum_{i\in N} \ell_{ij} \leq 1, \forall\, j \in N$.
6. $\sum_j \ell_{ij} \leq m_{\max}, \forall\, i, j \in N$.
7. $\ell_{ij} + \ell_{ji} \geq y_{ijc}, \forall\, i, j \in N, c \in C$.
8. $\ell_{ij} \leq \sum_{c\in C}(y_{ujc} + y_{jic}), \forall\, i, j \in N, c \in C$.
9. $Iy_c = \mathbf{b}_c, \forall\, c \in C$.
10. $\sum_{s=1}^{n_{\max}} z_{isc} \geq y_{ijc} + \ell_{ij} - 1, \forall\, i \in R, j \in N, c \in C \sum_{s=1}^{n_{\max}} z_{isc} \geq y_{jic} + \ell_{ij} - 1$.
11. $\sum_{s=1}^{n_{\max}} z_{isc} \leq 1, \forall\, i \in R, c \in C$.
12. $z_{isc} + z_{jsc'} \leq 1, \forall\, i \neq j \in R, s \in [1, n_{\max}], c, c' \in C$.
13. $\sum_{s=1}^{p} z_{isc} - \sum_{s=1}^{p} z_{jsc} \geq y_{ijc} + \sum_{s=1}^{n_{\max}} z_{isc} + l_{ji} - 3$, $\forall\, c \in C, i, j \in R, p \in [1, n_{\max}] \sum_{s=1}^{p} z_{isc} - \sum_{s=1}^{p} z_{jsc} \geq y_{ijc} + \sum_{s=1}^{n_{\max}} z_{jsc} + l_{ji} - 3$.

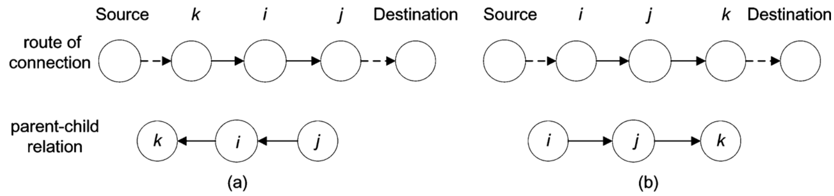


Fig. 3. Two cases considered for modeling precedence constraints.

14. $z_{isc} \geq g_{ijsc} \geq y_{ijc} + \ell_{ij} + z_{isc} - 2$, $z_{isc} \geq g'_{ijsc} \geq y_{jic} + \ell_{ij} + z_{isc} - 2$, $\forall\, i \in R$, $j \in N$, $s \in [1, n_{\max}]$, $c \in C$.
15. $\sum_c g_{ijsc}(b_c + O) \leq b_{\max}$; $\sum_c g'_{ijsc}(b_c + O) \leq b_{\max}$, $\forall\, i \in R$, $j \in N$, $s \in [1, n_{\max}]$.
16. $\sum_c g_{ijsc} \geq g_{ijs} \geq g_{ijsc}$; $\sum_c g'_{ijsc} \geq g'_{ijs} \geq g'_{ijsc}$, $\forall\, i \in R$, $j \in N$, $s \in [1, n_{\max}]$, $c \in C$.
17. $\sum_j g_{ijs} + \sum_j g'_{ijs} \leq m_{\max}$, $\forall\, i \in R$, $j \in N$, $s \in [1, n_{\max}]$.
18. $\sum_{i,j \in N} y_{ijc} \log(1 - p(i,j)) \geq \log(1 - p_c)$, $\forall\, c \in C$.
19. $z_{isc} s \leq \min\{n_{\max}, \lfloor l_c/SD \rfloor\}$, $\forall\, i \in R$, $s \in [1, n_{\max}]$, $c \in C$.

6) Placement Constraints: Constraint 1: End device nodes must be sited.

7) Routing Constraints: Constraint 2: An active link from i to j requires that nodes i and j be sited. Constraint 3: Parent-child relation is asymmetric (in tree topology). Constraint 4: End devices cannot be parents. Constraint 5: A node has at most one parent. Constraint 6: Each node has at most $m_{\max}$ children (limited by the number of guaranteed minislots per macroslot as set by selecting a ZigBee parameter). Constraint 7: Connection c is routed via node i to j, then either i is the parent or the child of j. Constraint 8: If i is the parent of j, then some connection is routed between i and j. Constraint 9: Each connection is routed, i.e., the balance equations hold.

8) Scheduling Constraints: Constraint 10: If j receives/sends data from/to its parent i for connection c, then i gets a macroslot for c. Further explanation of Constraint 10 is given below, after the listing of all the constraints. Constraint 11: For each connection, each node is assigned at most one macroslot. Constraint 12: Each macroslot is assigned to at most one node. Constraint 13: If i sends data to its parent/child j for connection c, which it receives/sends from/to its child/parent, then i's macroslot for c precedes j's macroslot for c. Further explanation of Constraint 13 is given below, after the listing of all the constraints. Constraint 14: Defines auxiliary variables g_{ijsc}/g'_{ijsc}—If parent i sends/receives data to/from node j for connection c, and has macroslot s, then $g_{ijsc}/g'_{ijsc} = 1$. Also, if $z_{isc} = 0$, i.e., macroslot s is not assigned to i for connection c, then $g_{ijsc}/g'_{ijsc} = 0$. Constraint 15: Data sent/received by node i to/from node j in a common macroslot s must fit the bit capacity of the minislot. Constraint 16: Defines auxiliary variables g_{ijs}/g'_{ijs}—If parent i sends/receives data to/from node j, and has macroslot s, then $g_{ijs}/g'_{ijs} = 1$. Also, if $\sum_c g_{ijsc} = 0$, i.e., i does not send/receive data to/from j in macroslot s for any connection, then $g_{ijs} = 0$, and similarly for the primed variables g'_{ijsc} versus g'_{ijs}. Constraint 17: The number of communications between parent i and its children in any single macroslot cannot exceed the number of guaranteed minislots $m_{\max}$.

9) Quality of Service (QoS) Constraints: Constraint 18: Success rate for a connection exceeds its specification. Constraint 19: Latency requirement is met for each connection.

Constraints 10 and 13 require additional discussion, which we present next. In a ZigBee network of tree-topology, communications in the beacon enabled mode are performed between the parent nodes and their children in the time-slots assigned to the parent nodes. Therefore, if a link between node i and j is used by a connection and i is the parent of j, then certain macroslot(s) should be assigned to i so as to allow i to communicate with j. This is captured by Constraint 10. Constraint 13 enforces the precedence among the routers for each connection. Here two cases, as shown in Fig. 3, are considered: router i must be scheduled before router j if a communication from i to j is needed by a connection, and either (i) i needs to collect data from a child before forwarding to its parent j [Fig. 3(a)] or (ii) j needs to forward data collected from its parent i to one of its children [Fig. 3(b)].

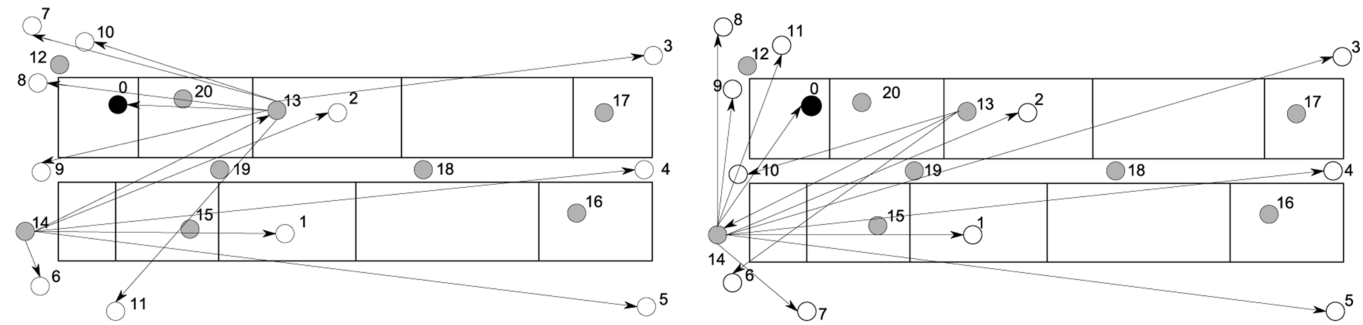
Remark 1: Although the above formulation considers the case when a round of communication fits a single beacon interval, there is nothing in the formulation that limits it to a single beacon interval: We can simply replace $n_{\max}$ with $n_{\max}N$, where N denotes the number of beacon intervals needed to fit one round of communication. The value of N can be found through a geometric search that tries the increasing powers of 2 as the choice for N.

IV. Implementation and Case Study

A. Application to Centralized/Distributed Estimation

The connection graphs associated with the two types of estimation algorithms are quite different. In the centralized case, the estimate is computed at a central node (also called gateway in this paper): The measurement values are directly sent from each node to the central node. In the distributed case, nodes in the network compute a fragment of the estimate locally, starting from their own measurements and fusing estimates from neighboring nodes. The number of the required connections in the distributed case is bigger than that of the centralized case, but the communication requirements are less strict since estimation is done by fusing information locally from many nodes, making the estimate result less sensitive to the packet loss.

Fig. 1 illustrates the input of the network synthesis [1(a) and 1(b) for centralized and distributed estimation, respectively]. The input comprises three sets of information: the nodes of the network, the communication requirements and the building geometry. The nodes represented by small circles in Fig. 1 can be of three different types: sensors (white circles) are sources of messages, gateways (black filled circle) are computing nodes which can be sources or sinks, and candidates for routers



(a) (b)

Fig. 4. Synthesized networks for centralized and distributed estimation.

(gray filled circles) that are intermediate nodes that relay the messages. The connection requirements are shown as directed edges, connecting a source node to a sink node. The last set of inputs is the building geometry. The input includes a two-dimensional floor plan of a building. Each wall is captured as a rectangle in the two-dimensional space.

B. Software Platform

We have built our prototype implementation of the proposed algorithm on the Communication Synthesis Infrastructure (COSI) [11] framework. While we reuse the COSI library to input the specification of the problem and for other input/output functions, we wrote the main module required to encode the optimization problem.

Keeping the building automation application in mind, the input to the communication synthesis tool is provided in a graphical format. The graphical interface is based on the SVG format and allows to describe the geometry of the building (i.e., the layout of the walls), the location of sensors, actuators, gateways and the connections among them. Each connection is annotated with the required QoS for that connection. In addition, the user can also specify the candidate locations for additional nodes (i.e., routers) that can instantiated by the optimization algorithm. The SVG description is parsed into an internal representation that includes a data structure to represent a network of components and a data structure to represent the building geometry. The building geometry is stored as a set of walls that are represented as surfaces in a three-dimensional space with an associated property of thickness.

The other input required by the COSI tool is a description of the available components that can be used to build the network. This description is provided by a separate XML file that captures a library of possible nodes. Each node can be either a full functional device (FFD) or a reduced functional device (as defined by the ZigBee standard). A node is characterized by parameters of performance and cost. The parameters are the maximum number of input and output links, the energy consumption for radio operation, the battery capacity, the installation and the maintenance cost (i.e., the cost associated with replacing the batteries). Given these two inputs, we use the COSI libraries to parse them and we then invoke our algorithm that generates the optimization problem presented in Section III-A as an instance of a pseudo-boolean optimization problem. We then use the SAT4J [1] solver to get an optimal solution which is used by COSI to generate a graphical representation of the network and a textual report.

C. Simulation Results

The **requirement**, in the case of centralized estimation, consists of 11 connections and 9 candidate router locations, as shown in Fig. 1(a). The latency requirement for each connection between a sensor and the central node is 1.5 s, a packet length is of 64 bits, an overhead is of 176 bits, and a maximum probability of error is required to be 10^{-4}.

The **parameters** of the objective function are as follows: $c_i = \$7002, \forall\, i$, $e_r = 132\,\text{nJ}$, $e_t = 236\,\text{nJ}$, $e_B = 30\,\text{KJ}$, $c_B = \$40$, $T_{\text{life}} = 20\,\text{years}$, and $BI = 0.49$ s. Based on the simulation results (Table I), we pick $SO = 3$, $BO = 5$, $n_{\max} = 2^{BO-SO} = 4$, $m_{\max} = 7$, and so $B_{\max} = 960\,\text{symbols} \times 4\,\text{bits/symbol} \times 2^{SO} = 31720$ bits, and $b_{\max} = B_{\max}/16 = 1920$ bits. With the above selected parameters, all the connections can be correctly routed and scheduled within one beacon interval. The corresponding synthesized network is shown in Fig. 4(a) in which the directed edges are used to illustrate the parent-child relations. The resulting synthesized network installs 2 routers (routers 13 and 14) and 13 links. It has the cost of \$15 994.

The corresponding router scheduling of the synthesized network is shown in Fig. 5(a). There are four macroslots labeled 1–4 (as indicated below each macrosolot), each assigned to either router 13 or 14 (as indicated above each macroslot). The shaded (resp., gridded) part of a macrosolot indicates communication from (resp., to) children nodes, whose node numbers have been indicated. The unshaded portions correspond to open minislots with no communication. The nodes within a shaded/gridded region can be scheduled in any order and so their minislot assignments are not shown. The content in the form of $s \rightarrow d$ within the parenthesis following each node number indicates the connection serviced by that node as an intermediate node. The connections from sensors 7, 8, 9, 10, and 11 to the gateway (indexed 0) are done in macroslot 1 via their parent, router 13. In macroslot 1, router 13 gets the message from the gateway for the connection to sensor 1. In macroslot 2, router 13 communicates with its parent, router 14, to further transmit the message collected from the gateway in the previous macroslot to sensor 1, and to obtain the messages from sensors 4, 5, and 6. Thereby, in macroslot 2, the connection from the gateway to

TABLE I
SIMULATION RESULTS FOR VARIOUS $n_{\max}$, $m_{\max}$

$n_{max} = 8$	Centralized			Distributed		
	$m_{max} = 15$	$m_{max} = 10$	$m_{max} = 7$	$m_{max} = 15$	$m_{max} = 10$	$m_{max} = 7$
number of variables	40413	40413	40413	80877	80877	80877
number of constraints	98543	98543	98543	221459	221459	221459
cost ($)	4371	8006	8117	5271	8980	9204
running time (s)	11	?	18291	34	13754	4096
number of routers installed	1	2	2	1	2	2
$n_{max} = 4$	Centralized			Distributed		
	$m_{max} = 15$	$m_{max} = 10$	$m_{max} = 7$	$m_{max} = 15$	$m_{max} = 10$	$m_{max} = 7$
number of variables	22737	22737	22737	45489	45489	45489
number of constraints	54299	54299	54299	120263	120263	120263
cost ($)	8623	15772	15994	10422	17721	-
running time (s)	7	19695	12738	21	21071	1990
number of routers installed	1	2	2	1	2	-

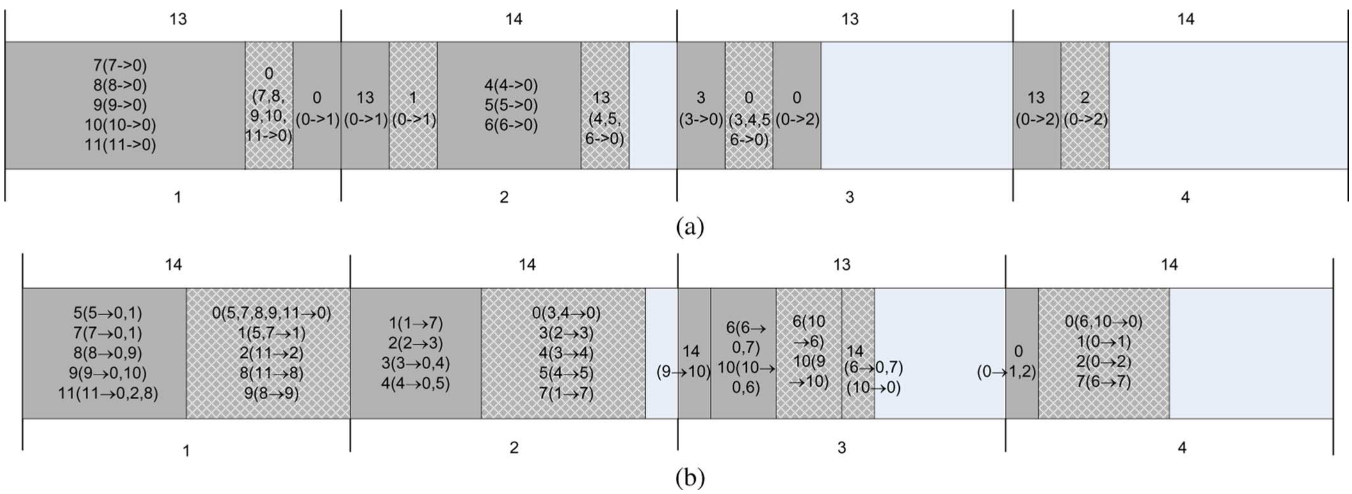


Fig. 5. Scheduling results for centralized and distributed estimation.

sensor 1 is completed. Next, in macroslot 3, the message sent by sensor 3, together with the ones from sensors 4, 5, and 6 received in macroslot 2 is delivered downstream to the gateway through router 13. Also, router 13 gets the message from the gateway. This message is further transmitted to sensor 2 in macroslot 4 via router 14. In this manner, the connections from sensors 3, 4, 5, and 6 to the gateway are accomplished in macroslot 3 and the connection from the gateway to sensor 2 in macroslot 4.

In the distributed estimation case, the **requirement** consists of 23 connections and 9 candidate router locations, as shown in Fig. 1(b). The **parameters** of the simulation are as follows: The communication among the neighbors happens with a period of 1 s which is also their maximum delay. Communication with the central estimation point happens every 2 s, which is the corresponding maximum delay. We pick $m_{\max} = 10$ (the other parameters remain the same as in the centralized case) by taking into account the tradeoff between performance and computation (see Table I). The synthesized network for the distributed estimation is, as shown in Fig. 4(b). The synthesized network installs 2 routers (routers 13 and 14) and 13 links, and its cost is $17 721. Note a higher cost compared to the centralized case is expected since more than twice the number of connections need be supported.

The scheduling of the synthesized network is given in Fig. 5(b), where macroslots 1, 2, and 4 are assigned to router 14 and macroslot 3 is assigned to its parent, router 13. The connections from sensors 5, 7, 8, 9, and 11 to the gateway, the connections from sensors 5 and 7 to sensor 1, and the connections from sensor 11 to sensors 2 and 8 are completed in macroslot 1 via their parent, router 14. In macroslot 1, router 14 also receives the message from sensor 9 for the connection to sensor 10. Similarly, the connections from sensor 1 to 7, 2 to 3, 3 to 4, 4 to 5, and sensors 3, 4 to the gateway are completed in macroslot 2. Next, in macroslot 3, the message sent by sensor 10 and that sent by sensor 9 in macroslot 1 and relayed by router 14 in macroslot 2 are further delivered to sensor 6 and 10 respectively through router 13. Also, router 13 gets the messages from sensors 6, 10, which are eventually transmitted to the gateway and sensor 7 via router 14 in macroslot 4. In addition, the connections from the gateway to sensors 1 and 2 are completed in macroslot 4.

Care should be taken while picking the values for $n_{\max}$ and $m_{\max}$ of a given set of connections specification: A small $m_{\max}$ increases computation burden as combination of multiple routers that must be explored increases complexity for routing and scheduling. Similarly, a large $n_{\max}$ makes computation for optimization more involved, while a small $n_{\max}$ may cause the problem infeasible. The optimization results for various choices for $n_{\max}$ and $m_{\max}$ are as shown in Table I, in which "—" denotes "no solution obtained" (since SAT4J is unable to decide whether there exists a solution to the given ILP formulation) and "?" denotes "no complete solution returned" (since SAT4J is unable to terminate within 15 hours although it can find certain solutions).

V. Conclusion and Discussions

It is greatly desirable that automated methods be developed for the synthesis of embedded networks that are cost-effective and correct-by-construction. The synthesis problem (focusing on ZigBee networks) was formalized as an Integer Linear Programming in this paper. Decisions were made for router placement, connection routing and scheduling. We also applied the proposed approach to the synthesis of wireless networks for centralized and distributed estimation in building automation applications, and presented a software implementation as well as simulation results for a practical-sized building automation network.

For the cases that the problem size (number of variables and constraints) is prohibitive, a hierarchical decomposition approach may be pursued to improve the scalability of the proposed synthesis formulation.

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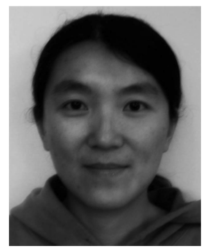

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